The Octagon Puzzle

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Answer: 12

There are a few ways of going about solving this puzzle. The most straightforward method is to add up rows until you get to 3216.

The first row must have 8 x 50 seats in it and the second 8 x 47, the third 8 x 44 etc. To save time you may notice that the number of seats in each row are decreasing by 24 each time.

I made a table on the right to keep track of the total seats. It's clear to see there that we require 12 rows to get exactly 3216 seats.

Row number	Seats in that row	Cumulative seats
1	400	400
2	376	776
3	352	1128
4	328	1456
5	304	1760
6	280	2040
7	256	2296
8	232	2528
9	208	2736
10	184	2920
11	160	3080
12	136	3216

Taking it further…

This is another excellent example of an arithmetic series (first featured in the handshake puzzle solution). If this puzzle were to be significantly larger then the straightforward method above would be quite arduous. Fortunately, Carl Friedrich Gauss's formula for the sum of an arithmetic series can solve any such puzzle with a little algebra:

a	First term
d	Common difference
n	Number of terms
S	Sum of all n terms

$$S = \frac{n}{2}(2a + (n-1)d)$$

In this puzzle we know a, d and S and need to work out n. Can you use this formula to calculate the answer?

There is also a proof showing why Gauss's formula works and it is beautiful.